

Bar Modelling for Secondary

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Pictorial

Bar modelling is a visually powerful, pictorial approach to representing mathematics and problem-solving.

Typically, maths teachers see it as a tool to help low/middle attaining students access more complex maths due to its simplicity. This is only partially true.

High attaining students often dislike pictorial strategies. This can leave them unable to articulate a developed understanding of a mathematical idea beyond the algorithm they used in the lesson.

Bar modelling is my favourite pictorial strategy, as it's incredibly transferrable between different domains of maths. Students who engage with pictorial strategies such as bar models have better long-term retention of topics.

<p>Ahmed and Brandon share £30 in the ratio 4:1. How much does each person get?</p>	<p>Ahmed and Brandon share some money in the ratio 4:1. Ahmed gets £30. How much does Brandon get?</p>
<p>Ahmed and Brandon share some money in the ratio 4:1. Ahmed gets £30 more than Brandon. How much does each person get?</p>	<p>Ahmed and Brandon share some £30 in the ratio 4:1. Ahmed then gives Brandon £4. What fraction of the money does Brandon now have?</p>

Misconceptions/caveats

Bar modelling is not meant to replace the mathematical chain of reasoning. It serves as a bridge to students grasping it. The calculations that students do should still be written out formally.

You don't have to programme every bar model laboriously into your slides. Most of the time, I draw them by hand, either using the board or the visualiser.

Make sure that you have routines around bar modelling. For high-attaining students, have a minimum number of questions that they have to use a bar model for, so that they see the relatedness between the topics you show them.

Bar models are not just for word problems and proportionality, they can be applied to anything that involves the 4-operations.

Further information

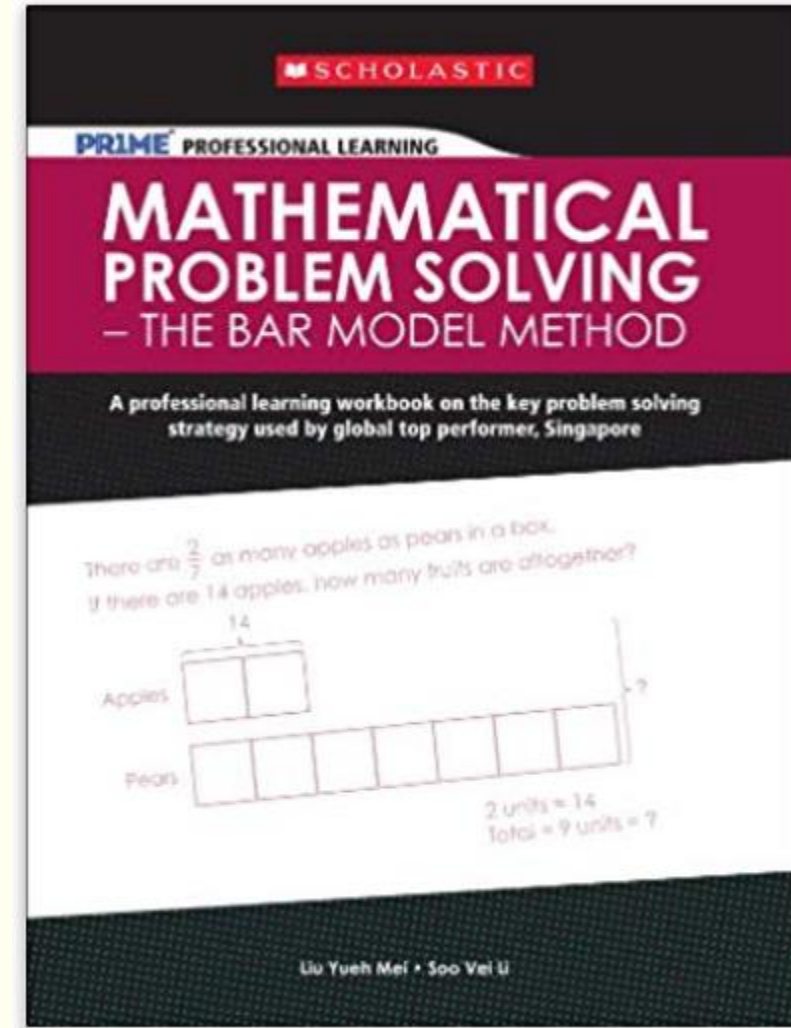
[Mathematical problem solving](#)

[Thinking blocks](#)

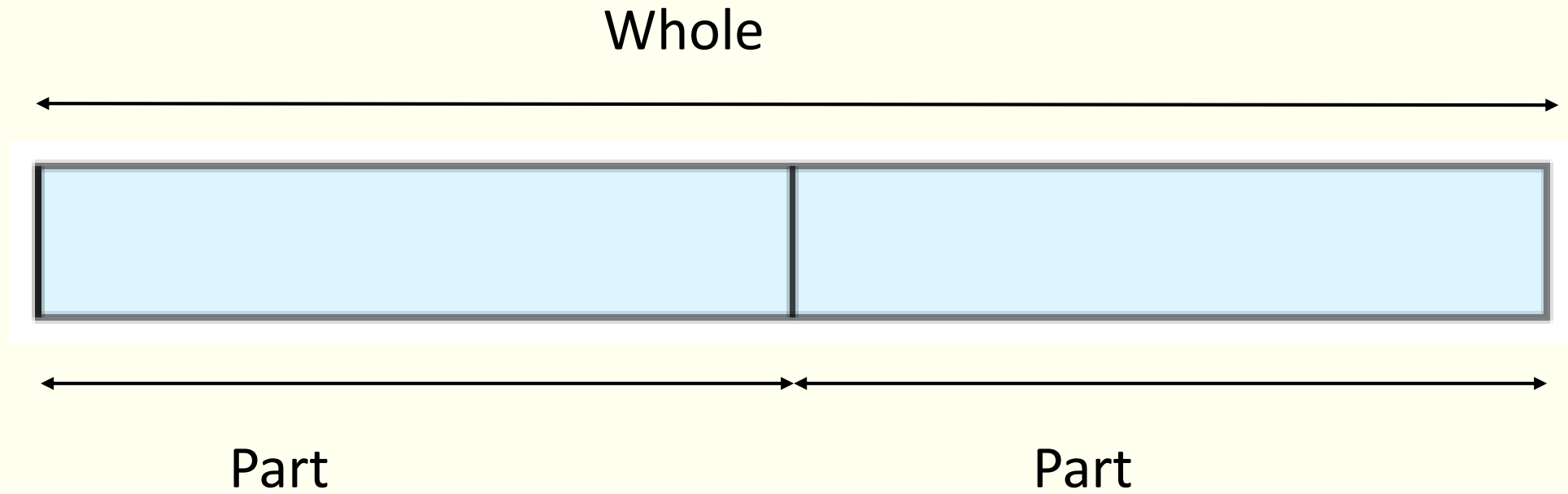
[Mathsbot](#)

[Simultaneous equations twitter thread \(@emaths\)](#)

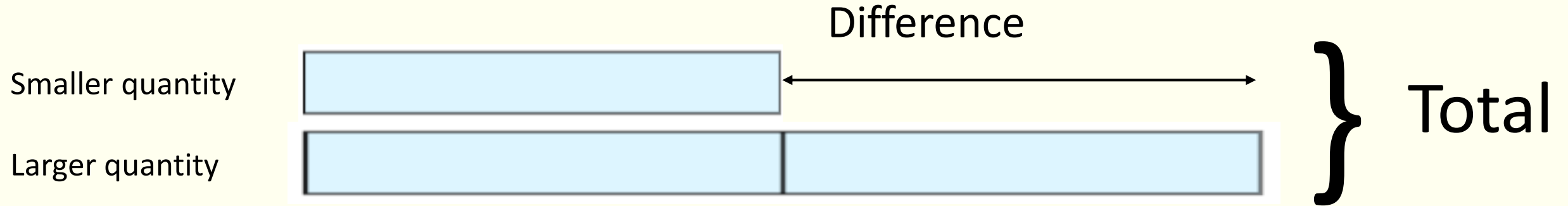
[Mr Barton podcast – Paul Rowlandson](#)



Basics – parts and wholes

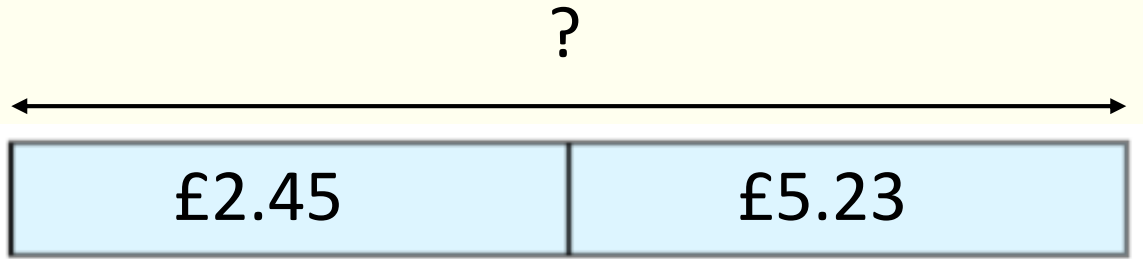


Basics – comparison model



Word problems

Annie has £2.45, Billy has £5.23. How much do they have together?



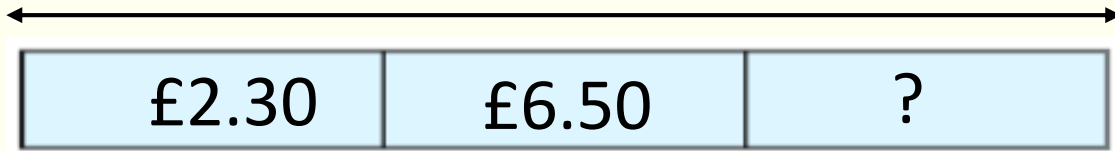
$$£2.45 + £5.23 = £7.68$$

Charlie has £6.50, Dave has £13.70. How much do they have together?

Word problems

Billy has £12.50. He spends £2.30 on sweets and £6.50 in the chip shop. How much does he have left?

£12.50



? £8.80

$$£2.30 + £6.50 = £8.80$$

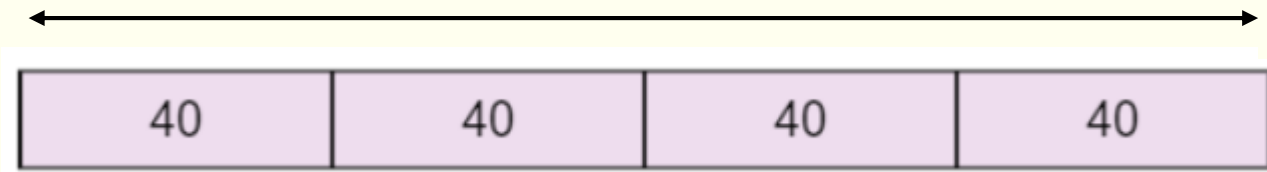
$$£12.50 - £8.80 = £3.70$$

Andy gets £20 in pocket money. He spends £1.20 on sweets, £4.50 on a t-shirt, and £5 on a dvd. He saves the rest. How much does he save?

Division problems

4 friends win £160. They share the money equally. How much money does each person get?

£160

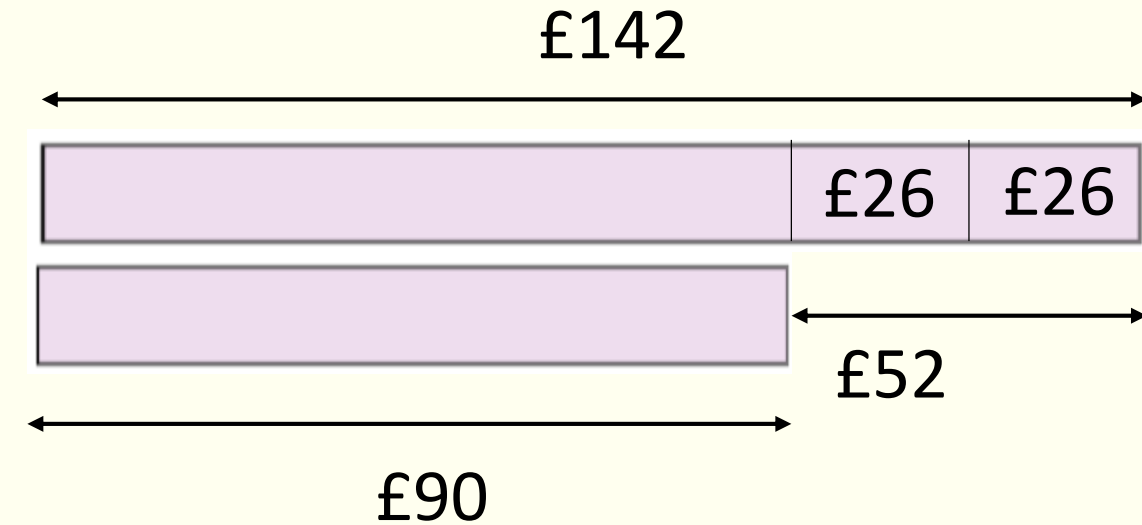


$$£160 \div 4 = £40$$

6 friends go to a restaurant. Their bill is £150. They split it equally. How much does each person pay.

Word problems

Bill has £142, John has £90. Bill gives John some money so that they have the same amount each. How much will they each have?



$$£142 - £90 = £52$$

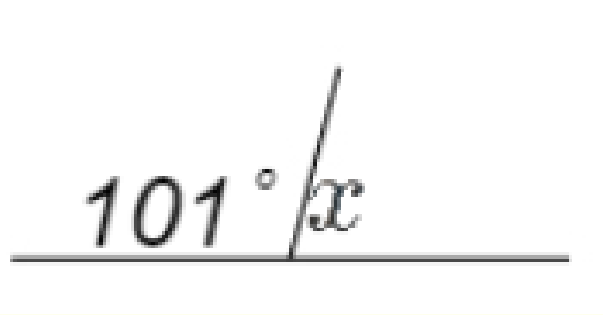
$$£52 \div 2 = £26$$

$$£90 + £26 = £116$$

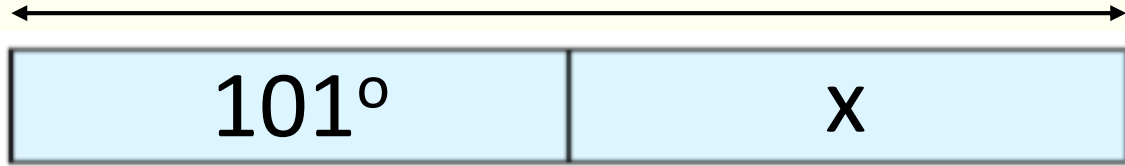
Bill has £152, John has £80. Bill gives John some money so that they have the same amount each. How much will they each have?

Bill has £142, John has £37. After they spent an equal amount of money, Bill has 4 times as much money as John. How much did they each spend?

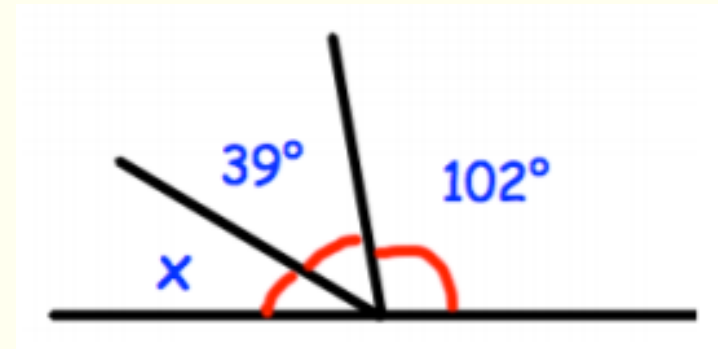
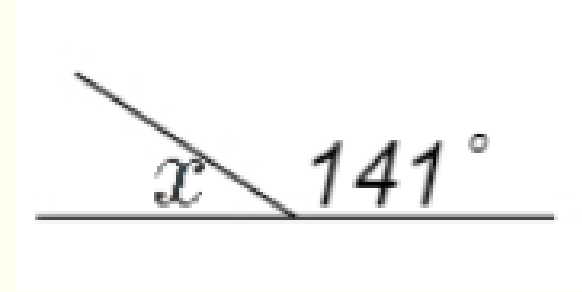
Angles

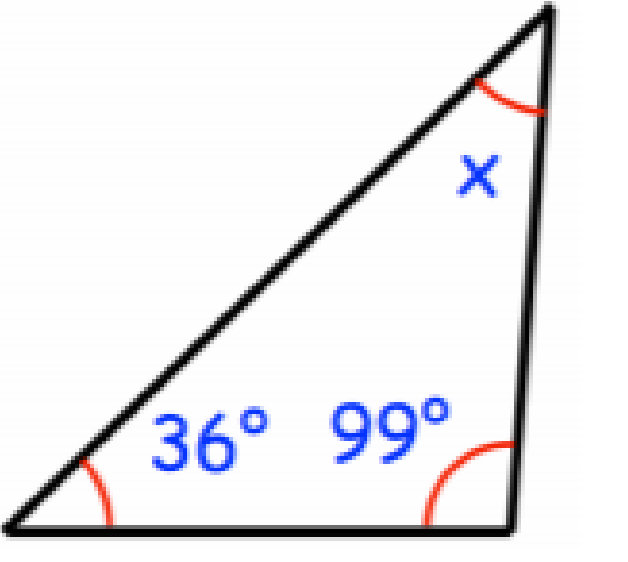


180°

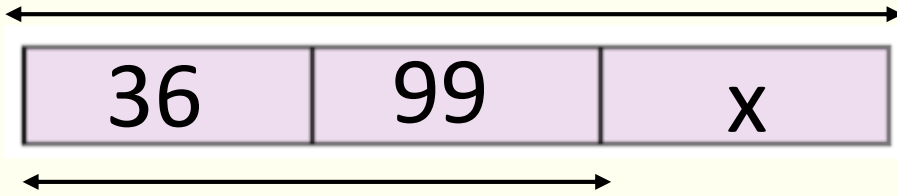


$$180^\circ - 101^\circ = 79^\circ$$





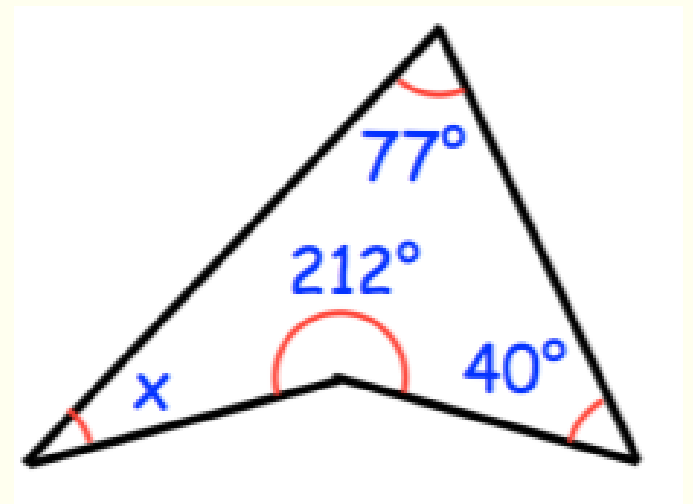
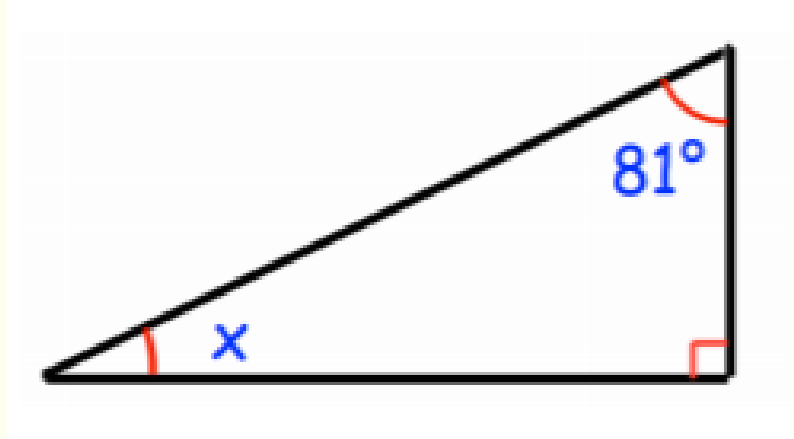
180°

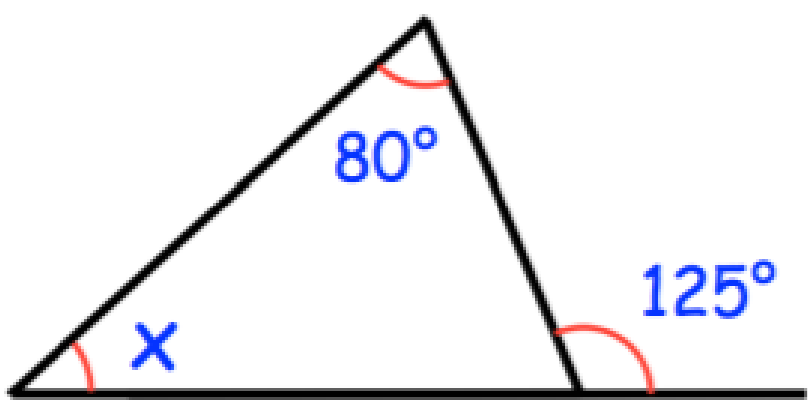


? 135°

$$36^\circ + 99^\circ = 135^\circ$$

$$180^\circ - 135^\circ = 45^\circ$$





180°

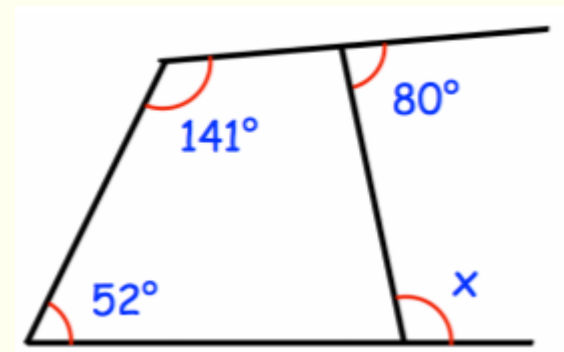
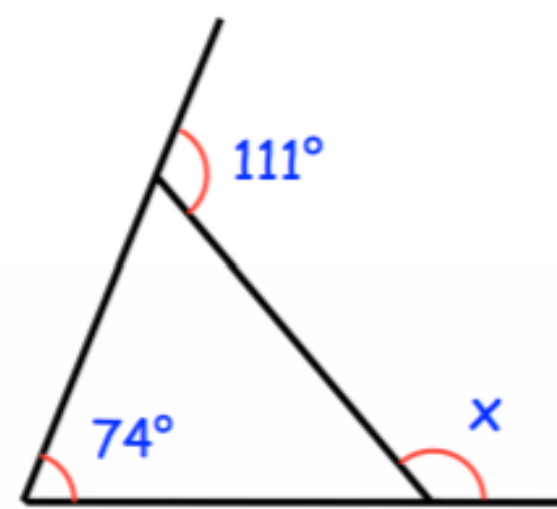
x	80	55	125
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180°

$$180 - 125 = 55$$

$$80 + 55 = 135$$

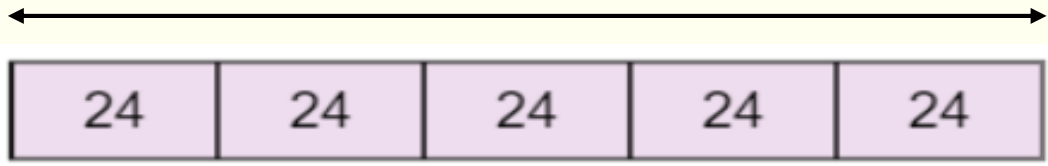
$$180 - 135 = 45$$



Fractions

Find $\frac{2}{5}$ of 120

120



?

$$120 \div 5 = 24$$

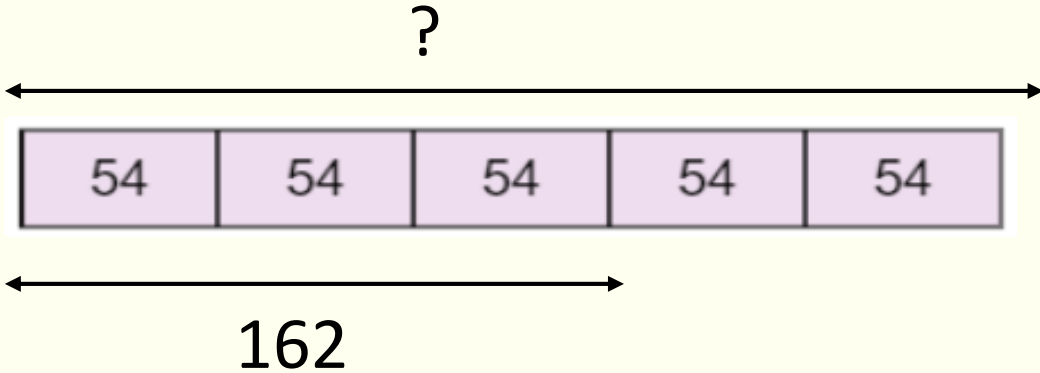
$$24 \times 2 = 48$$

Find $\frac{5}{6}$ of 120

Find $\frac{5}{6}$ of $\frac{1}{2}$

Reverse Fractions (AQA 2017 1H)

$\frac{3}{5}$ of a number is 162. What is the number?



$$162 \div 3 = 54$$

$$54 \times 5 = 270$$

$\frac{2}{5}$ of a number is 182. What is the number?

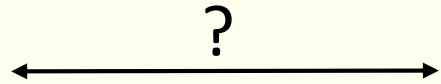
$\frac{2}{5}$ of x is 182. x is $\frac{5}{8}$ of y .
What is the value of y ?

Fractions

Dave has £200. He spends $\frac{3}{8}$ of his money on trainers.

He spends $\frac{2}{5}$ of what's left on a new squash racket. How much does he have left over?

200



$$200 \div 8 = 25$$

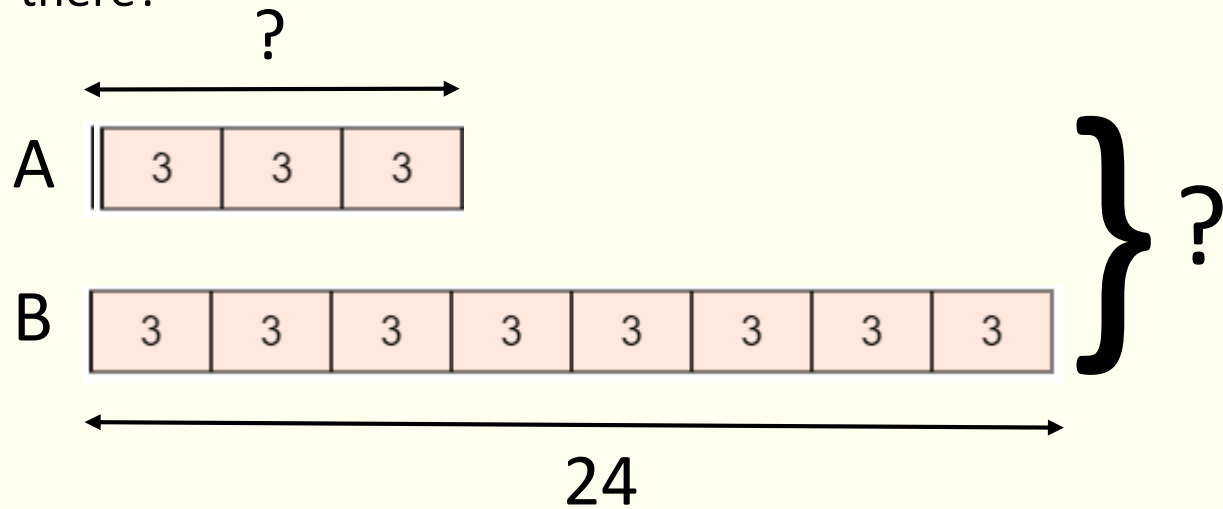
$$25 \times 3 = 75$$

Dave has £200. He spends $\frac{1}{4}$ of his money on trainers.

He saves $\frac{2}{3}$ of what's left. How much money does he save?

Fractions

There are $\frac{3}{8}$ as many apples as bananas in a box. If there are 24 bananas, then how many pieces of fruit are there?



$$24 \div 8 = 3$$

$$24 + 9 = 33$$

There are $\frac{5}{6}$ as many apples as bananas in a box. If there are 24 bananas, then how many pieces of fruit are there?

There are $\frac{3}{8}$ as many apples as bananas in a box. If there are 24 apples, then how many pieces of fruit are there?

Ahmed and Brandon share £30 in the ratio 4:1. How much does each person get?

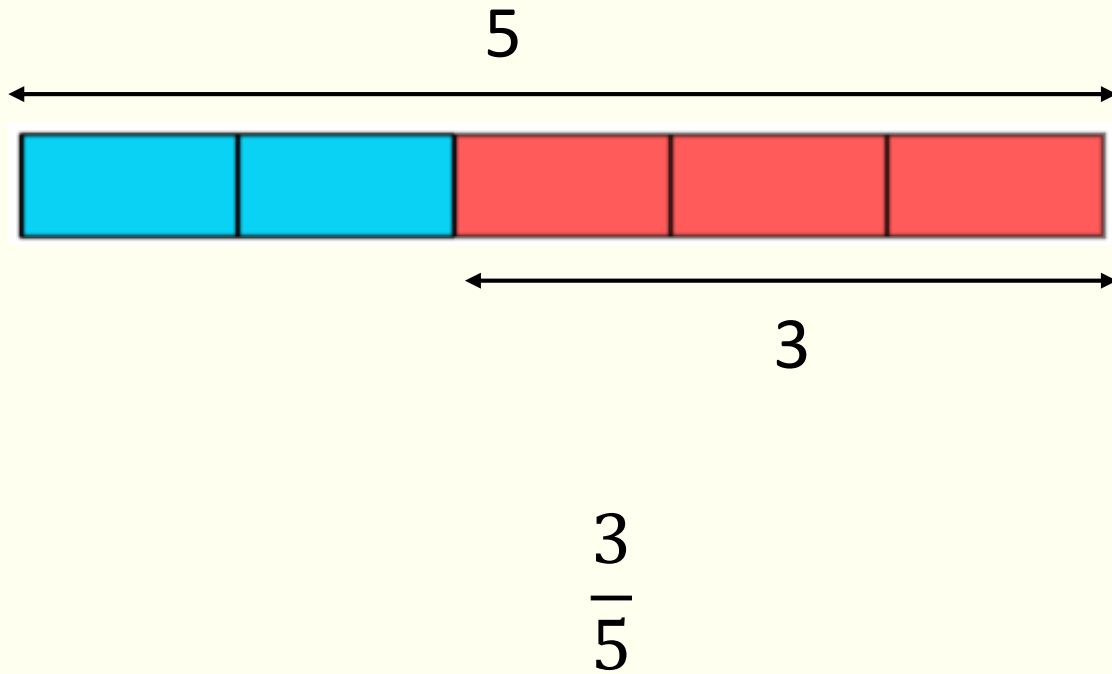
Ahmed and Brandon share some money in the ratio 4:1. Ahmed gets £30. How much does Brandon get?

Ahmed and Brandon share some money in the ratio 4:1. Ahmed gets £30 more than Brandon. How much does each person get?

Ahmed and Brandon share some £30 in the ratio 4:1. Ahmed then gives Brandon £4. What is the ratio do they now share the money in?

Ratios as fractions

To make purple paint, blue and red paint are mixed in the ratio 2:3. What fraction is red?



To make purple paint, blue and red paint are mixed in the ratio 2:5. What fraction is red?

$$x:y = 2:5$$

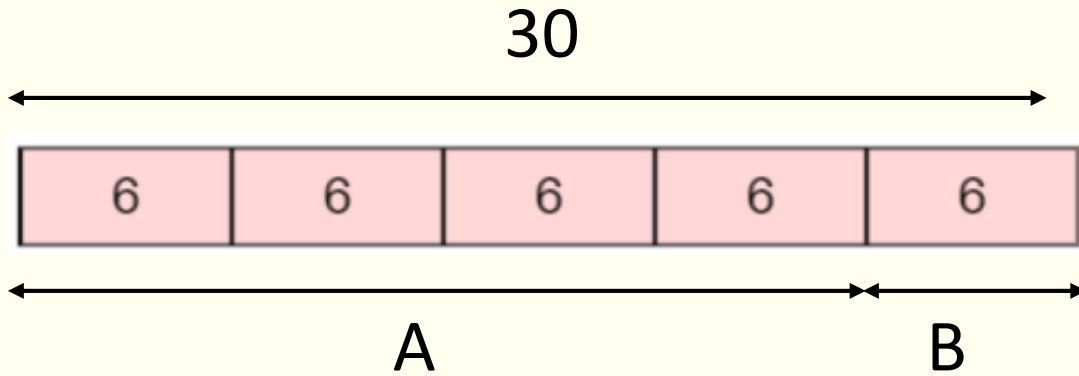
What fraction of y is x ?

What fraction of x is y ?

What fraction of the total is x ?

Ratios

Ahmed and Brandon share £30 in the ratio 4:1.
How much does each person get?



$$4 + 1 = 5$$

$$30 \div 5 = 6$$

$$A: 4 \times 6 = 24$$

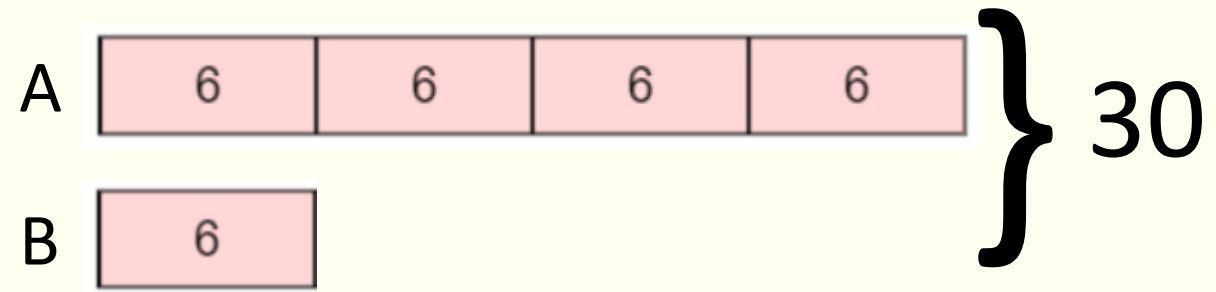
$$B: 1 \times 6 = 6$$

Ahmed and Brandon share £35 in the ratio 4:3.
How much does each person get?

$$x + y = 88$$

$$x:y = 7:4$$

Work out the value of $x - y$



$$4 + 1 = 5$$

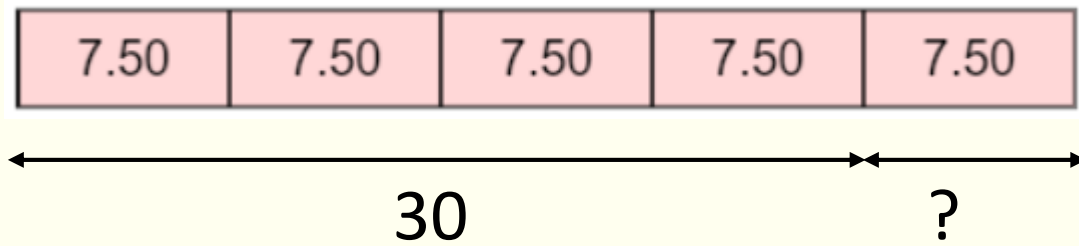
$$30 \div 5 = 6$$

$$A: 4 \times 6 = 24$$

$$B: 1 \times 6 = 6$$

Ratios

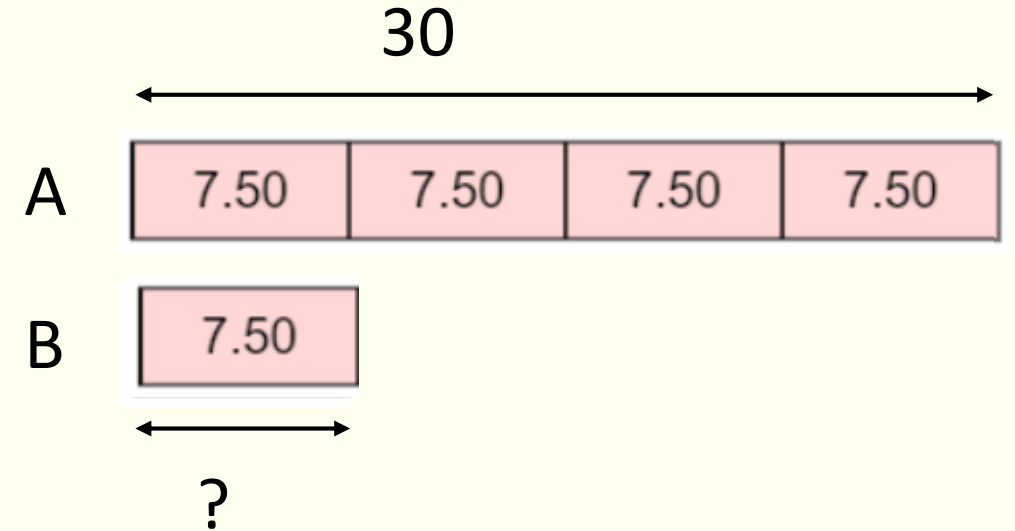
Ahmed and Brandon share some money in the ratio 4:1. Ahmed gets £30. How much does Brandon get?



$$30 \div 4 = 7.5$$

B: £7.50

Ahmed and Brandon share some money in the ratio 4:5. Brandon gets £30. How much do they have together?

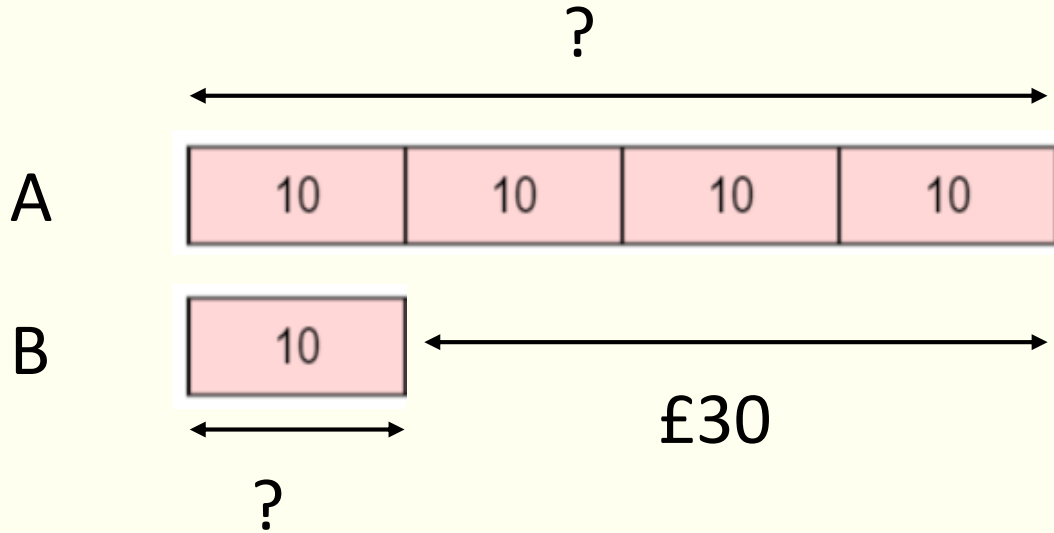


$$30 \div 4 = 7.5$$

B: £7.50

Ratios

Ahmed and Brandon share some money in the ratio 4:1. Ahmed gets £30 more than Brandon. How much does each person get?



$$4 - 1 = 3$$

$$30 \div 3 = 10$$

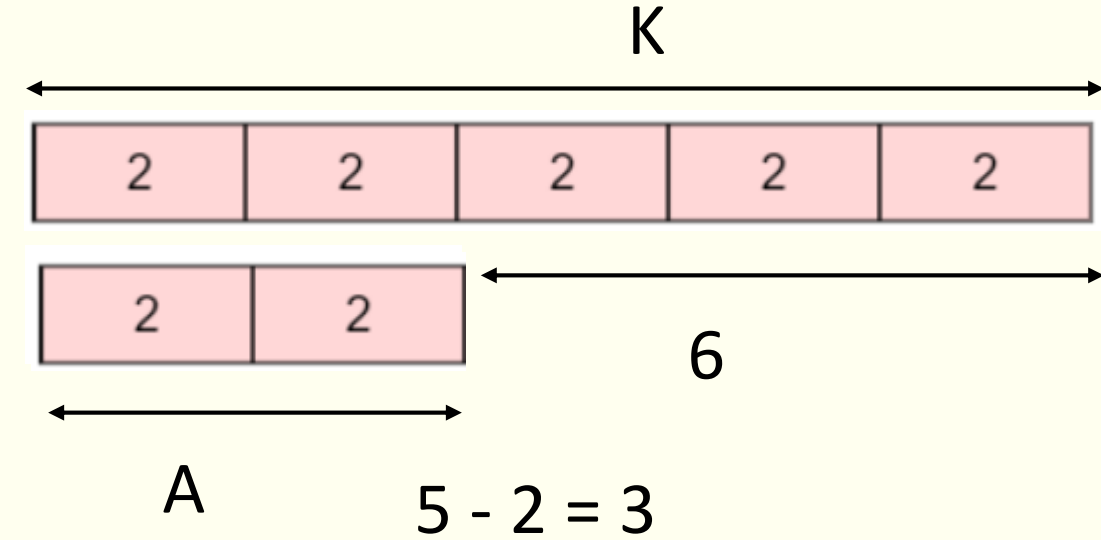
$$A: 4 \times 10 = 40$$

$$B: 1 \times 10 = 10$$

Ahmed and Brandon share some money in the ratio 7:5. Ahmed gets £30 more than Brandon. How much does each person get?

Ratios

Kyle is 6 years older than Alfie. In 3 years time, the ratio of Kyle's age to Alfie's age will be 5:2. What age is Kyle now?



$$5 - 2 = 3$$

$$6 \div 3 = 2$$

$$K: 5 \times 2 = 10$$

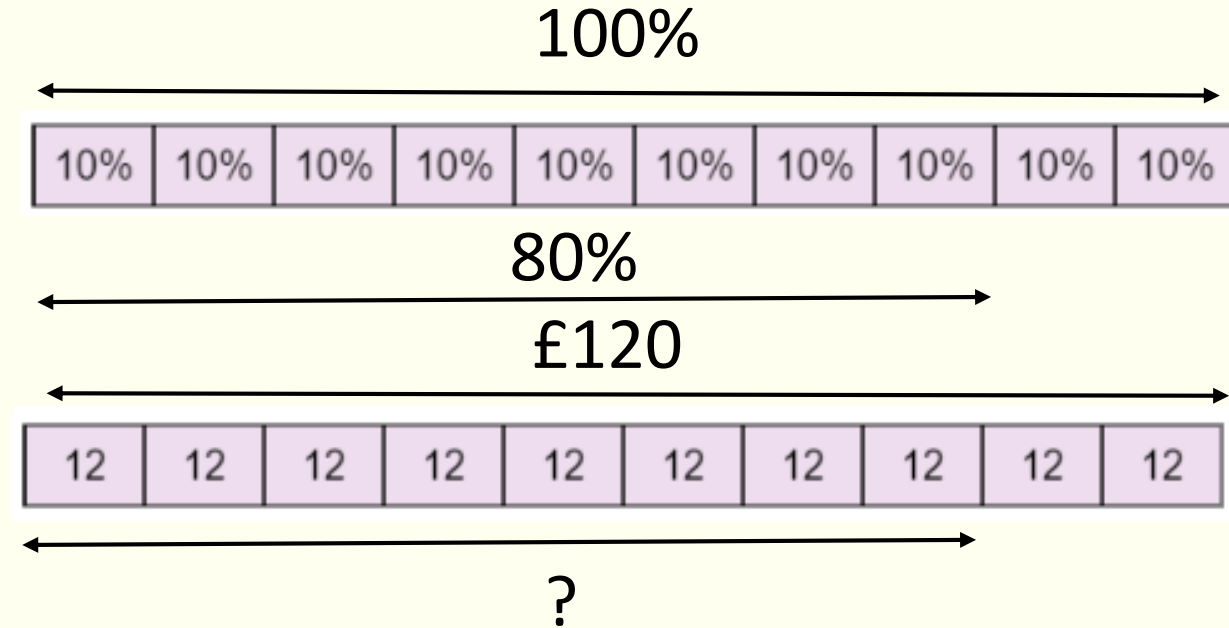
$$B: 2 \times 2 = 4$$

$$10 - 3 = 7$$

Kyle is 10 years older than Alfie. In 3 years time, the ratio of Kyle's age to Alfie's age will be 5:3. What age is Kyle now?

Percentages

Dave gets £120 pocket money. He spends 80% of it. How much did he spend?



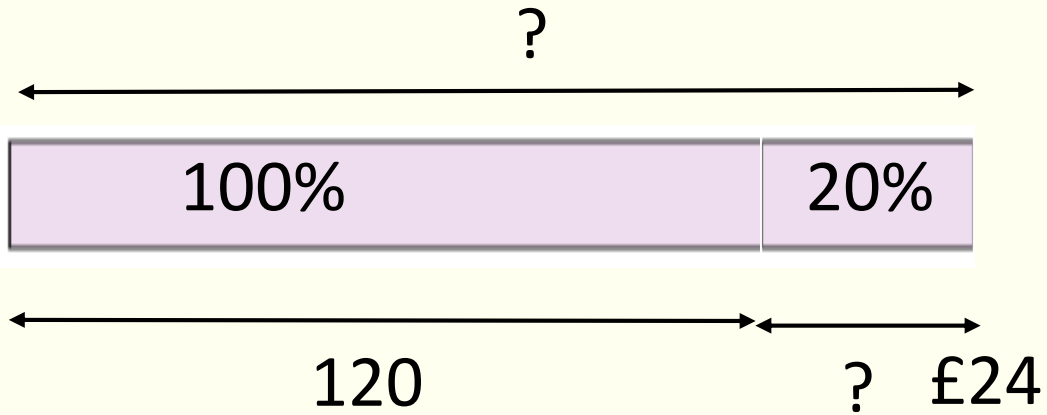
$$120 \div 10 = 12$$

$$12 \times 8 = \text{£}96$$

Dave gets £110 pocket money. He spends 70% of it. How much did he save?

Percentages

Dave gets £120 pocket money. His parents increase this by 20%. How much pocket money does he get now?



$$120 \div 10 = 12$$

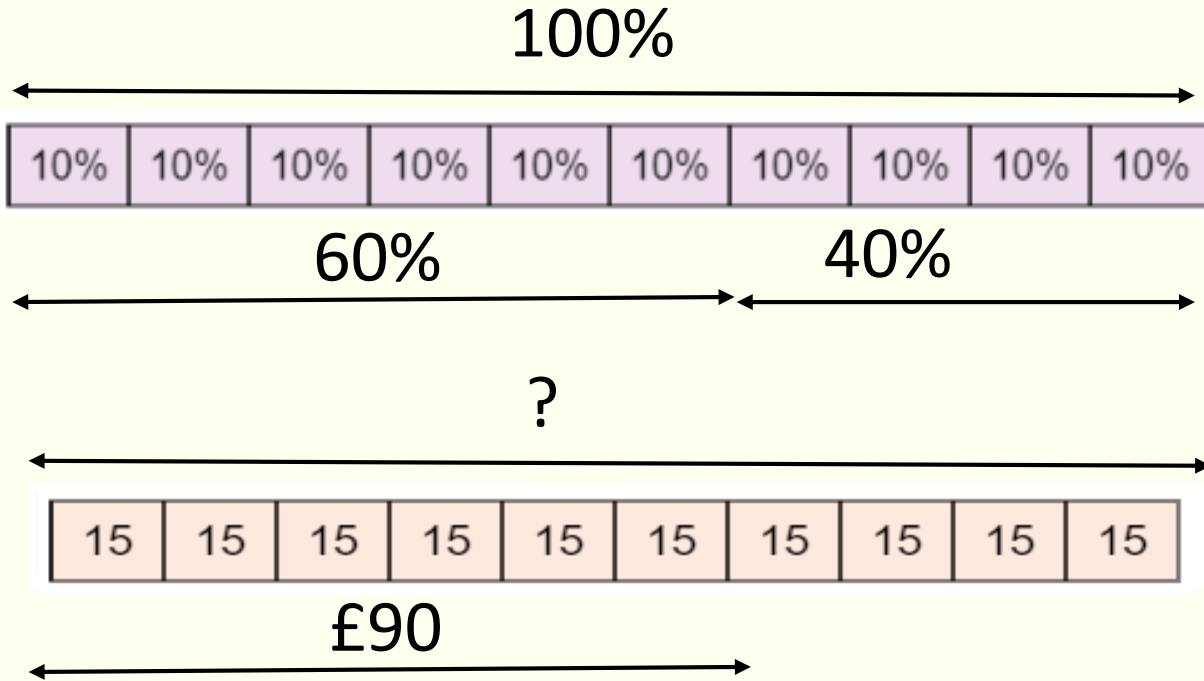
$$12 \times 2 = \text{£}24$$

$$12 \times 2 = \text{£}120 + \text{£}24 = \text{£}144$$

Dave gets £120 pocket money. After a visit to Ready to Learn, his parents reduce this by 70%. How much pocket money does he get now?

Percentages

A dress is reduced in a sale by 40%. It now costs £90. How much did it cost before the reduction?



$$90 \div 6 = 15$$

$$15 \times 10 = 150$$

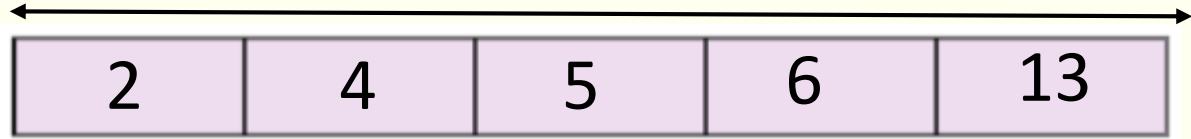
A dress is reduced in a sale by 20%. It now costs £120. How much did it cost before the reduction?

Due to Brexit, a dress increases in price by 20%. It now costs £150. How much did it cost before the reduction?

Mean

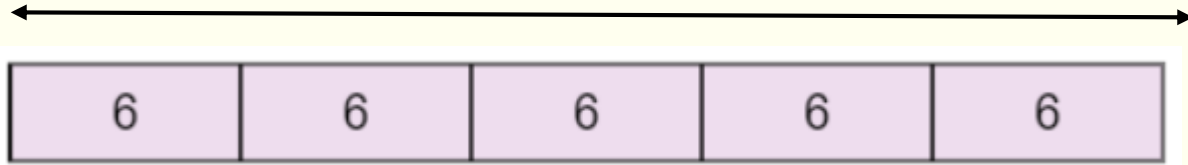
Find the mean of 2, 4, 5, 6, 13

30



$$2 + 4 + 5 + 6 + 13 = 30$$

30



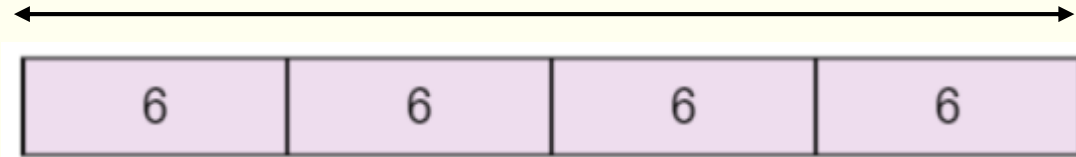
$$30 \div 5 = 6$$

Find the mean of 2, 4, 5, 6, 8

Reverse Mean

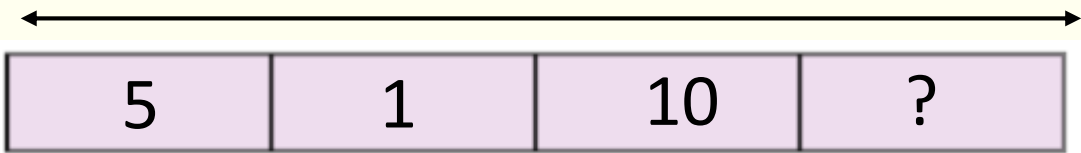
5, 1, 10, ? Mean = 6

24



$$6 \times 4 = 24$$

24



?

$$5 + 1 + 10 = 16$$

$$24 - 16 = 8$$

6, 2, 11, ? Mean = 6

. Adam and six other men ran a race.

The times, in seconds, of the six other men are shown.

9.75 9.79 9.80 9.88 9.94 9.98

The mean time for **all seven** men was 9.83 seconds.

Did Adam win the race?

You must show your working.

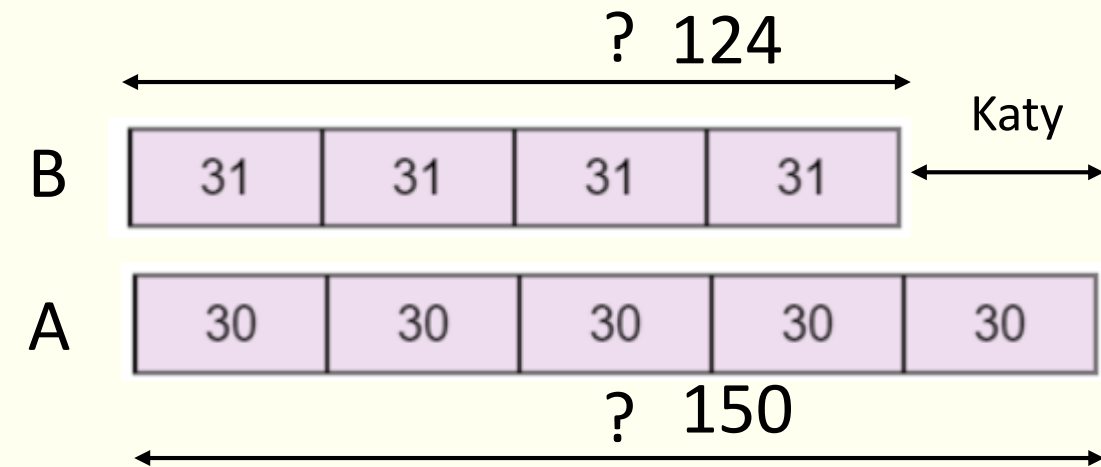
Reverse Mean

. At a nursery, the mean age of 4 children is 31 months.

Katy joins the nursery.

The mean age of all 5 children is now 30 months.

Work out the age of Katy.



$$31 \times 4 = 124$$

$$150 - 124 = 26$$

$$30 \times 5 = 150$$

. Walkden Reds is a basketball team.

At the end of 11 games, their mean score was 33 points per game.

At the end of 10 games, their mean score was 2 points higher.

Jordan says,

"Walkden Reds must have scored 13 points in their 11th game."

Is Jordan right?

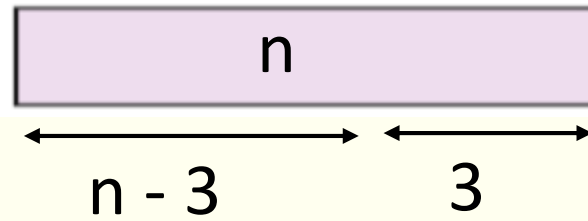
You must show how you get your answer.

Expressions

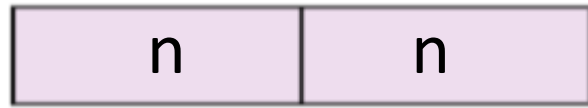
3 more than n



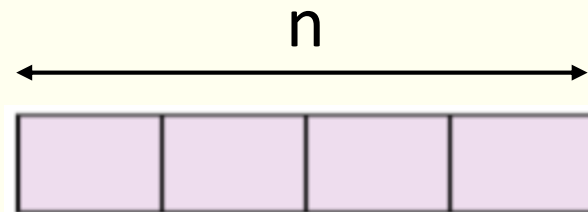
3 less than n



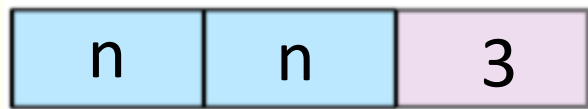
Double n



A quarter of n



3 more than double n



10 more than n

10 less than n

Quadruple n

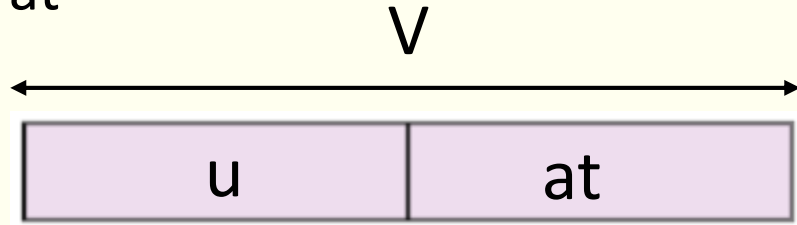
A sixth of n

3 more than quadruple n

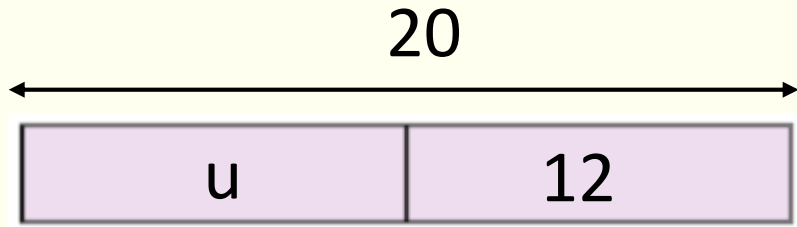
Quadruple 3 more than n

Formulae

$$V = u + at$$



If $V = 20$, $a = 3$ and $t = 4$, find the value of u



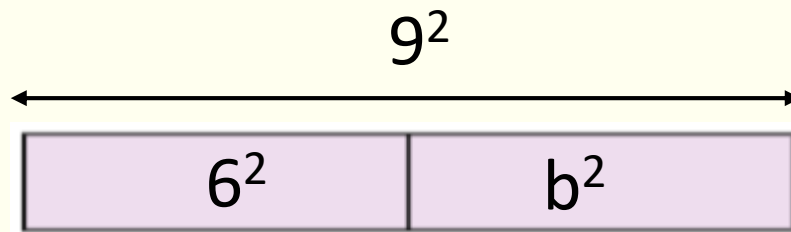
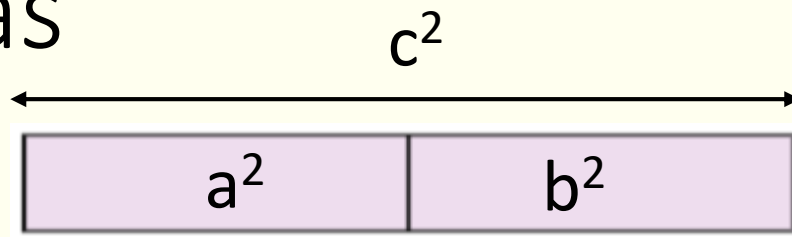
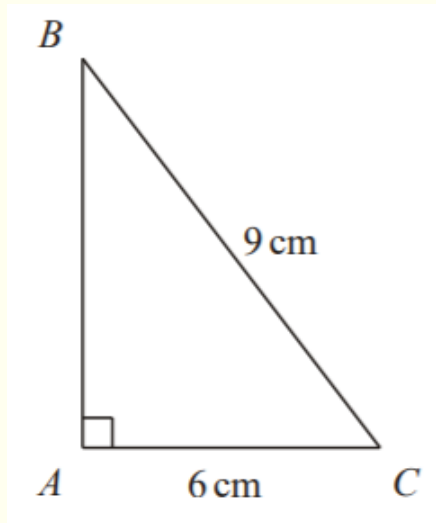
$$V = u + at$$

If $V = 30$, $a = 3$ and $t = 6$, find the value of u

$$V = u + at$$

If $V = 30$, $a = 4$ and $u = 2$, find the value of t

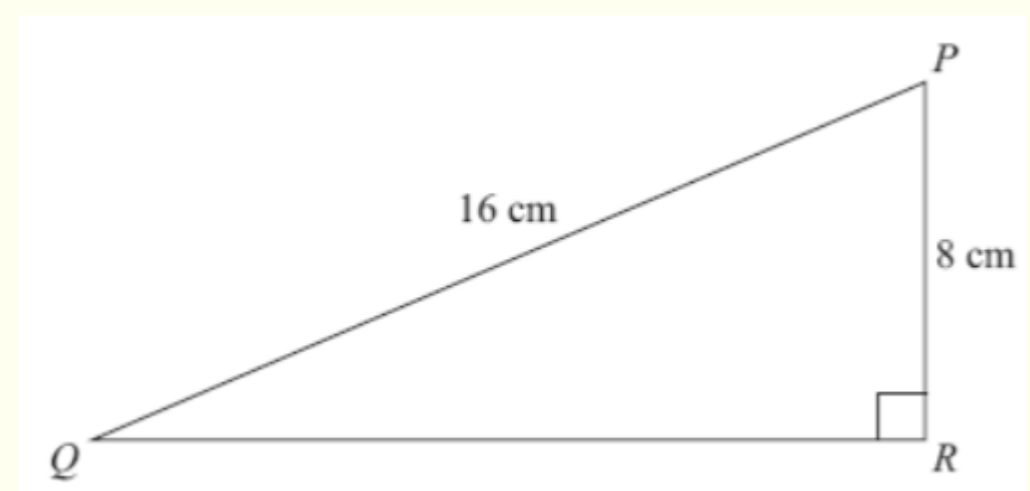
Pythagoras



$$6^2 + b^2 = 9^2$$

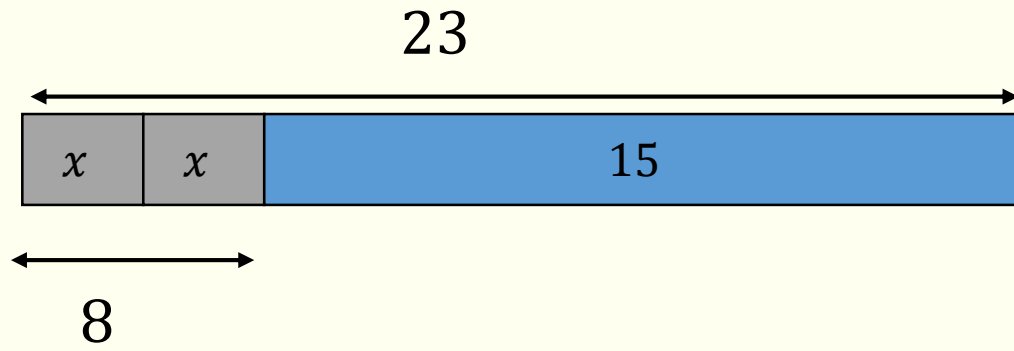
$$b^2 = 9^2 - 6^2$$

etc



Solving equations

Solve $2x + 15 = 23$



$$2x + 15 = 23$$

$$\boxed{-15} \quad \boxed{-15}$$

$$2x = 8$$

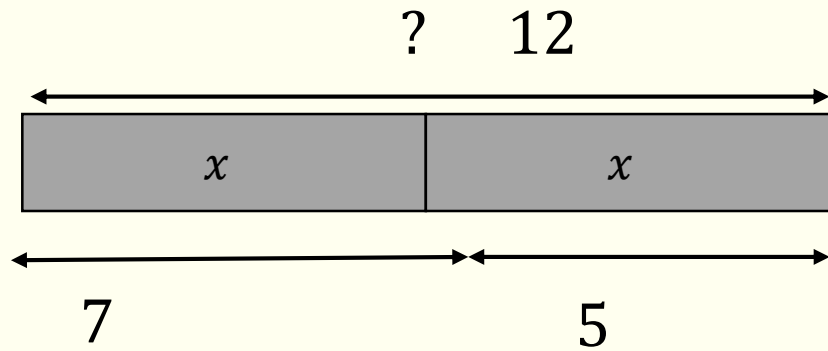
$$\boxed{\div 2} \quad \boxed{\div 2}$$

$$x = 4$$

Solve $4x + 15 = 23$

Solving equations

Solve $2x - 5 = 7$



$$2x - 5 = 7$$

+5

+5

$$2x = 12$$

$\div 2$

$\div 2$

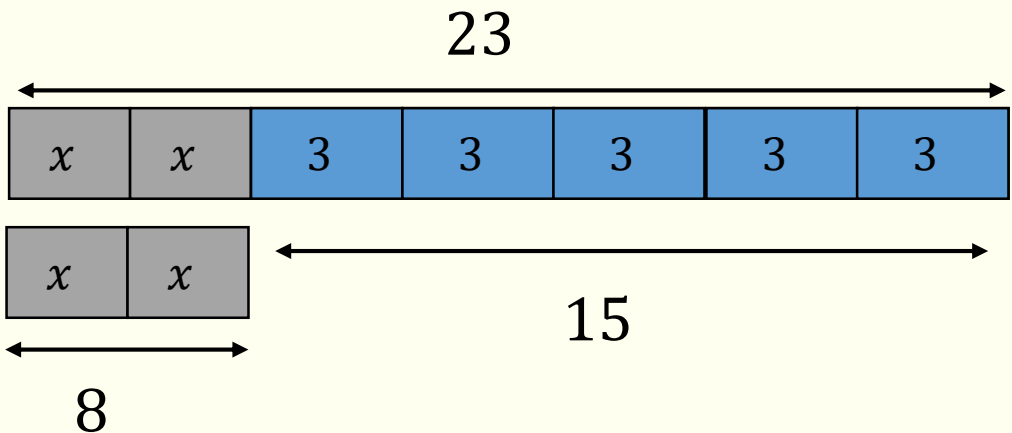
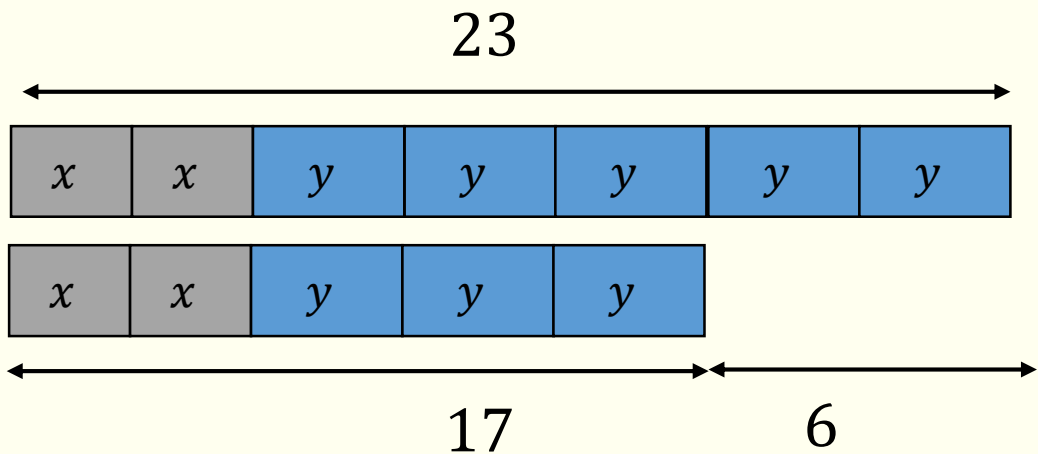
$$x = 6$$

Solve $4x - 5 = 7$

Simultaneous equations (Pixi Maths)

Solve: $2x + 5y = 23$

$2x + 3y = 17$



Solve: $2x + 5y = 23$

$2x + 3y = 17$

$$\frac{2y}{2} = \frac{6}{2} \quad y = 3$$

$2x + 5y = 23$

$2x + 15 = 23$

$-15 \quad -15$

$2x = 8$

$x = 4$