



***FMSP***

*Let maths  
take you further*



# *Problem solving*



# *Warm up*

What number, when multiplied by itself,  
is equal to  $27 \times 147$ ?

“Mathematical practice is concerned with finding solutions to problems, whether related to practical problems (applied) or internal to mathematics (pure). Indeed, mathematical research is itself a form of problem-solving.

Problem-solving is therefore an important, if not the most important, component of our discipline.”

Teaching Problem Solving in Undergraduate Mathematics  
(Badger, Sanguin, Hawkes et al 2012)

“The ability to solve problems is at the heart of mathematics. Mathematics is only ‘useful’ to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name ‘problem solving’.”

Cockcroft Report, 1982

# *Categorising problems*

- Four broad ‘types’ of problem (Askew and Wiliam 1995)
  - Standard problems
  - Non-standard problems
  - ‘Real world’ problems
  - Puzzles

## Standard problems (word problems)

stories requiring translation into mathematical questions.

*Biscuits come in packs of 6. A carton holds 24 packs. How many biscuits are there in a carton?*

## Non-standard problems

situations for which the problem solver will not have a set routine or procedure for finding a solution.

*There are 20 guests at a party. If everyone shakes hands with everyone else, how many handshakes will there be in total?*

## ‘Real world’ problems

situations where the students will have to select the relevant information and set up a mathematical model that simplifies the situation.

*How can we improve the queuing system at dinnertime?*

## Puzzles

problems depending upon luck, insight or unusual strategies for their solution.

*A square piece of cheese can be cut into 9 smaller squares with four cuts – two across and then two down, If the pieces are re-arranged between cuts, can the 9 smaller squares be made with fewer cuts?*



# *Solving unstructured problems*

When given problems a lot of students expect to be told what to do.

We want students

- to be able to think their way into a problem
- to understand that it is important to have a try
- to know that there can be several different ways of tackling many problems
- not to always expect to be able to get everything perfect at the first attempt

## *What challenges are there for developing students' problem-solving skills?*

- Many students expect to be told what to do rather than think about a problem for themselves
- As a teacher how do you offer support with this without doing the thinking for the student?

# *Supporting students to develop problem-solving skills*

- Ask your students about the sort of mathematical ideas that might be needed in a problem
- Get students to talk about what they intend to do to solve the problem

# *Supporting students to develop problem-solving skills*

- Use open questions rather than closed questions that require a single fixed response
- Keep asking students ‘why?’
- Develop a culture where mistakes are seen as something to learn from rather than something to be ashamed of. Mistakes are an important part of problem-solving

# *Some techniques*

- students work in small groups
- allow time for them to think about and discuss how they might start a problem
- ask 'open' questions that encourage a range of responses
- encourage students to explain their reasoning (most find this really hard)
- try to get students to see mistakes as something to be learned from

# Problem

What number, when multiplied by itself,  
is equal to  $27 \times 147$ ?

## Things to do with students

- Explore the relationship between prime factors and divisors
- Investigate the prime factorisations of square numbers
- Ask questions like ‘The prime factorisation of 63 is  $3 \times 3 \times 7$ . The prime factorisation of 10 is  $2 \times 5$ . What is the prime factorisation of 630?’

The FMSP produces resources for teaching, enrichment and the promotion of mathematics.



Home > Resources > Problem Solving Resources

## Problem Solving Resources

### GCSE Problem Solving Resources

The FMSP produces problem solving resources for GCSE students and teachers. The majority of these can be found on the **GCSE problem solving page** of the KS4 Resources section of this website. Here is a taste of our GCSE problem solving resources.

#### Problem Solving Business Cards

The FMSP has produced a set of business card size problems for use with GCSE problems. You can get the individual problems from your Area Coordinator or download them here:

<p>What fraction of the large square is shaded?</p>		<p>I have 3 tins of mushroom soup, 3 tins of baked beans and 3 tins of rice pudding in my cupboard, but the labels have all fallen off. If I pick 3 tins for my lunch, what different combinations of food could I have? (Assume that I will only open the tin just before cooking and eating so I could eat them in any order!)</p> <p>Extension: I tell my friend about my 9 tins and she brings over three tins of mushy peas, also without labels and we mix them up – how many different meals could I have if I chose three tins from the 12?</p> <p>What is the pattern for any number of tins?</p>	<p>A farmer has 240m of fencing and wants to create an enclosed area for his sheep. What is the biggest area that he can fence off?</p> 
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# *FMSP problem-solving resources*

- These are not designed as practice exam questions but are intended to help learners develop the mental flexibility they will need for the new elements of the exam
- They could be embedded into a scheme of work or they could form the basis for a student problem-solving event
- They focus on pure problem-solving rather than functional contexts.



- Now have a go

# *Experience of problem-solving with sixth formers*

Weekly problem-solving class for STEP:

- To begin with students find this very hard
- After a few weeks one or two students start to make breakthroughs
- Other students feed off this
- By about 5 weeks in most students are enjoying the sessions and looking forward to the classes.

# *The importance of problem selection*

Giving students the right question at the right time is important

- A good question for a student is
  - a) one that they can access given their current confidence and ability levels, but can't immediately see the solution
  - b) one that is sufficiently challenging for them to feel a sense of satisfaction from making progress with it.

# The Further Mathematics Support Programme

Our aim is to increase the uptake of AS and A level Further Mathematics to ensure that more students reach their potential in mathematics.

The FMSP works closely with school/college maths departments to provide professional development opportunities for teachers and maths promotion events for students.

To find out more please visit  
[www.furthermaths.org.uk](http://www.furthermaths.org.uk)

